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Discussion of

"BEHAVIOR OF STRUCTURES SUBJECTED TO A FORCED VIBRATION"

by Charles T. G. Looney
(Proc. Sep. 451)

C. T. G. LOONEY,[1] A.M. ASCE.—Mr. Chang has shown the similarity between the solution for natural frequencies and that for the buckling of a continuous structure. Both of these problems can be solved by means of simultaneous linear equations similar to slope-deflection equations. It is of importance to point out this similarity between the problems because of the numerous examples of the solution of continuous structures by slope-deflection. A word of caution is justified with regards to the general applicability of this method. A structure with many members results in a large number of simultaneous equations. A large number of simultaneous equations is difficult, if not impossible, to solve. Mr. Chang mentions Professor T. F. Hickerson's solution of indeterminate frameworks by the use of fixation factors. This method of analysis is directly applicable to the solution of natural frequencies and forced vibrations. The use of fixation factors eliminates convergence difficulties.

Mr. Conwell emphasizes the importance of presenting results of analytical studies. In the field of vibration the profession needs a background of theoretical studies and correlation with experience by practicing engineers. Mr. Conwell's criticism of the criterion for natural frequencies of a structure in the vicinity of the natural frequencies of a member is justified from a practical point of view. The proof that certain frequencies exist is of primary importance. This criterion does give a point at which to start investigation. Usually a sketch such as that shown on Fig. 5A will give a close value. The starting value of 1.4 used in the example of Fig. 6 was obtained in this manner. The suggestion of a range of frequencies is excellent. In cases where the members differ in their relative frequencies and stiffnesses the natural frequencies of the structure will usually lie near to the natural frequencies of the members; a range of one below and one above the values of $f/f_1 = 1, 4, 9, 16$ ..... is suggested.

Mr. Conwell's confirmation of the computation of natural frequencies by the Rayleigh-Ritz Method is a valuable contribution. This method is undoubtedly to be preferred. A background of information on the modes of vibration for structures, such as is shown for a continuous beam on Fig. 3, is a necessity. This again confirms Mr. Conwell's introductory remarks.

The author thanks Mr. Conwell for pointing out the influence of the several parameters and the illustrations of their separate influence shown on Table 7.

The author's choice of equal length and moment of inertia was to avoid changing the vertical scale of Fig. 5 and thus simplify the explanation.

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Discussion of
"FATIGUE IN RIVETED AND BOLTED SINGLE LAP JOINTS"

by J. W. Carter, K. H. Lenzen and L. T. Wyly
(Proc. Sep. 469)

J. W. CARTER,[a] A.M. ASCE, K. H. LENZEN,[b] A.M. ASCE and L. T. WYLY,[c] M. ASCE.—The authors are grateful to Mr. Woodruff, Mr. DeVries and Mr. Prince for the interest they have shown in this paper and for the number of pertinent questions they have raised.

Mr. Woodruff asks two questions regarding the AAR hanger investigation:

1) Why is the number of failures so small in comparison to the number which did not fail?
2) Why are there not failures reported in the adjacent truss diagonals?

The hanger failures involved were all riveted joints. Mr. Woodruff accepts the authors' conclusions that a high clamping force reduces the probability of slip and reduces the magnitude of slip, if it occurs, and that it also reduces the stress and strain concentration around the rivet hole. It has been pretty well established that clamping force may be expected not only to vary with grip but also from rivet to rivet among a group in a given joint where all have the same grip. Table 4 of Source 4 listed in Fig. 16 shows that in a group of six hot driven rivets 3/4" in diameter and having a grip of 1-3/16 in. the measured clamping force varied from 3,000 lbs. to 14,000 lbs. The percentage of rivets showing a low clamping force is small as compared with those showing a high clamping force. For the larger grips the clamping force was higher and the minimum value of clamping force was much higher. It has also been established that the hot driven rivets do not fill the holes. Professor Wilson's measurements (See Fig. 8 of Source 1 listed in Fig. 16) show clearances between the rivets and the metal at the sides of the holes varying from 0 to .055 in. for hot driven rivets 1" diam. and 5 in. grips. In general the shorter grip rivets may be expected to more nearly fill the holes and a few of them may be expected to show low clamping force.

Since in most cases the rivets may not be expected to initially fill the holes, slip will be necessary to produce a bearing stress. If the clamping force is high, such slip is less likely to occur and if it does occur, the stress and strain concentration resulting is much less severe.

The fact that relatively few hangers out of the total number in service have been reported as failing is entirely consistent with the above explanation and indeed what would be logically expected. It is good evidence that the great majority of the rivets in the hanger connections do have a high clamping force.

The explanation of the lack of failures in the truss diagonals adjacent to the hangers is, in the opinion of the authors, quite direct and logical.

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They believe that it is due to two factors:

1) The range of stress in the diagonals is more favorable to fatigue strength than in the hangers.
2) The number of cycles of maximum stress is much less in the case of the diagonals than in the case of the hangers.

As an illustration the case of the 120 ft. through truss span of Ref. 1, Fig. 3, p. 474; Fig. 6, page 476; Fig. 14, p. 482; and Fig. 15, p. 483 may be considered. There were reported 22 failures in the hangers in 15 identical bridges of this design, all at the upper connection of the hangers.

The beneficial effect upon fatigue strength of reduction in the stress range is well known. A somewhat simplified picture of this relation is given by the Goodman-Johnson diagram.[32] In the case of the U_1-L_1 hangers the range of stress is from a minimum of about 1.5 ksi D to a maximum of about 19.5 ksi D+L+I, each stress being computed at the failure section and including axial stress plus bending due to floorbeam deflection. The range of this stress is thus about 18 ksi for each repetition of maximum loading. In case of the adjacent diagonals U_1-L_2 the dead load is about 16% of the total D+L+I and the range of stress is somewhat less. Thus the stress range favors a little higher fatigue strength.

The hanger in the bridge in question receives at least two cycles of maximum loading every time the engine or a train crosses the bridge while the diagonal receives only one such cycle of maximum loading. Other things being equal, it would therefore require at least twice as many years to cause fatigue failures in the diagonals as in the hangers. In the cases of structures carrying some of the modern heavy car loads the hangers may get still more maximum loading cycles during the passage of a train.

Mr. Woodruff's suggestion, frequently expressed by others, that an unequal stress distribution across the hanger section may be the explanation, is not borne out by the facts. There is bending in the hanger, due to its rigid connection with the floorbeam and the deflection of the latter under load. But the bending stress is a maximum at the top of the floorbeam and is less than 40 as large at the failure section at the upper gusset. See Fig. 3 of Ref. 1. Also of the 22 failures reported, 7 occurred first at the inside angle of the hanger (nearest the center of track) where the bending would reduce the combined stress in the hanger. Furthermore, when all the 79 failures reported in riveted trusses are checked for this bending, it is found that 45 failed at the inside angles. Also when the bending due to secondary stress in the plane of the truss is considered, it is found that 43 failures occurred on the side of the hanger where bending would reduce the combined stress. See Table 2 of Ref. 1.

Only three hangers failed at the top of the floorbeam and two of these are attributed to resonant vibration. The reason that the hangers did not fail at the point of greatest bending is thought to be the fact that a gusset or a reinforcing plate always was extended up the vertical for several rivets above the floorbeam which probably reduced the tendency of the rivets at the floorbeam connection to slip and come into bearing. In the case of the third failure which occurred at the top of the floorbeam, such a reinforcing plate was absent. See Fig. 1, p. 473; Fig. 4, p. 475; and Fig. 11, p. 478 of Ref. 1.

Regarding the flow in ductile steel where large local stresses and strains

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32. Fig. 303, p. 298, Resistance of Materials, 3rd Edition, F. B. Seely, John Wiley and Sons, Inc.

occur, the authors believe that the story is illustrated by their Figs. 2, 3 and 4. Where the yield point of the steel is reached locally the material will flow, as shown in Fig. 2, but the stress concentration, i.e., the ratio of stress at the side of the hole to stress at the gross section, remains practically constant from this point on while the strain concentration increases as the loading is increased. See Fig. 4. It is important to remember also that at any stage of loading the member so that the steel at the point of consideration (here, the side of the hole) is stressed well above the elastic limit, a certain amount of rest or a few cycles of stress to this same prior high value will restore elasticity to the material. This explains why the fatigue cracks invariably are initiated at the point of maximum principal (tensile) stress and their trajectory follows that of the principal stress, all computed on an elastic basis. See Fig. 11.

The location and path of the fatigue crack is thus a direct clue to the source of the stress producing the crack. Where the crack begins about a sixth of the diameter above the center of the hole on the side of the rivet bearing and follows a curved path, as shown in Figs. 1(a) and 11, it is prima facie evidence that the principal tensile stress producing the crack is directly connected with the rivet bearing. Without such bearing, the crack would begin at the side of the hole on a diameter normal to the axis of the member, and would propagate in a straight line at right angles to the axial stress, as many tests have shown. In all of the few cases of actual fatigue failures in actual structures in this type of connection which the authors have had the good fortune to inspect and where the cracks could be seen, they conformed to the above pattern.

It cannot be too strongly emphasized that attention must be focused upon the state of stress and the stress level at the very local, small, spot where the crack begins. It has often been remarked that a doubling of the area of the hangers would cost relatively little in the cost of the structure and would cure the difficulty. The authors see no reason to believe that this would necessarily have any beneficial effect at all.

Mr. De Vries makes some good critical comments on the photoelastic model studied. The plastic rivet did not fill the hole in the plastic plates after load was applied, due to elastic yielding. Prior to load the fit was good. On the other hand, as noted above, the steel rivet seldom fits the hole as well as the plastic model. Many measurements by Professor Wilson and others have established this. The authors see no reason to believe that fatigue cracks start while good clamping force is present in the rivet. When clamping force is lost, the rivet heads are not stress participating and they are not stress relieving. It should be stated, however, that the authors regard the conditions of test of the plastic models as representing a limiting condition. But they believe that this limiting condition is approached more closely in practice when failures occur than is generally realized.

The authors do not feel that any marked change in results would have resulted in the plastic model experiments for measuring the stresses in the top of the plate due to the bearing of the head or nut, if a washer had been used. The washer being thin, would have distributed the bearing load out radially, perhaps over a circular area having a diameter greater than that of the bolt circle by about twice the washer thickness, or less. One reason they think this to be true is that cracks in the plate occurred outside the hole, as shown in Fig. 1(c) when the clamping force was high and the washer was thin. See test results on Study 2 Graph in Fig. 7(a). When the clamping force is decreased, see Test 2(a) or when the washers are made thicker, see

Study 3, all of Fig. 7(a), the fatigue strength was raised or failure did not occur.

Mr. De Vries questions whether the picture of overloading of the end rivets shown in Fig. 9 is a true one. Similar conclusions, i. e., that the end rivets in a row may be overloaded 100%, were arrived at by Davis, Woodruff, and Davis in Ref. 10 from measurements on large steel joints. They also found some premature failures of rivets in the end holes. See Col. 29 in Table 9 of Ref. 10. Measurements of slip distribution between gussets and the member in a recent experiment tend to support this same thesis.[33a] Two large joints representing a wide flange hanger connected to gussets were tested. The connection consisted of four lines of seven connectors each. In the first joint rivets were used excepting that the last two holes at the edge of the gusset in each line were connected with high strength bolts. In this case, the slip at the end of the connection occurred under lower load and were much larger than the slips at the center of the connection. The other joint was connected entirely by high strength bolts. First major slips occurred at stresses well above working loads. The same relative slip distributions were observed as for the riveted joint, but the slips near the center of the connection were very small.

Mr. Prince raises five important specific questions. The authors will discuss them in chronological order.

The authors believe that the question of whether fatigue failures in the hangers are due to inadequate design of the members or to use of riveted connections is answered by the following considerations:

1) At the points of failure the axial plus bending stresses in the hangers are not large. Even at the points of greatest bending the stresses are not large.
2) At the points of failure little bending stresses are present. There is no evidence whatever of high stresses except at the sides of the rivet holes where the cracks occur.

The only exceptions to the above statements are the 9 hanger failures in the deck truss bridge shown in Fig. 2, p. 473; Fig. 5, p. 475; and Figs. 12 and 13, pp. 480 and 481 of Ref. 1. Due to the type of framing, the axial plus bending stresses in these hangers at the failure sections would amount to about 55 ksi if computed on an elastic basis. The service life of these hangers was about 24 years. Three spans were involved.

In the hangers which failed the ratio of thickness of the main material to that of the gussets varied from .75 to 1.0. In no case were the gussets thinner than the main material. No gusset failures were reported. No results were obtained indicating any significance for any particular ratio of main material to gusset material thickness.

Regarding Mr. Prince's third question, the authors strongly believe, on the basis of Study 4 of Fig. 7(a), that in members designed for fatigue loading, no drifting at all should be permitted either in the shop or in the field in holes in the last two transverse rows at the edge of main or splice material and they are of the opinion that only very light drifting should be permitted near the middle of the connection. They feel that the evidence now available indicates that drifting should not be used to induce stress in

33. J. W. Carter, J. C. McCalley, and L. T. Wyly, AREA Bull. 517, September-October 1954. (Reference to this footnote appears on page 682-7.)
33a. Figs. 20-23 incl. pp. 246-249 incl.

members which will carry fatigue loading.

Mr. Woodruff and Mr. Prince are both concerned regarding what slip may be expected in bolted joints and whether they will slip enough to allow distribution of load to take place between the various bolts. Tests on large joints reported by various investigators for the Research Council for Riveted and Bolted Structural Joints, including the papers in this symposium,[34, 35] as well as a recent report by two of the authors,[33] all show that slip in bolted joints will occur when the relations between load, clamping force and coefficient of friction permit it. Also much more slip can occur in a bolted joint than in a riveted joint, since much more clearance is provided between the bolts and the metal at the sides of the holes. Recent experiments by Professor Hechtman on the magnitude of total slip to be expected on a long time test will yield very valuable information. In the opinion of the writers more detailed study and experimentation is needed on this subject before a rational basis for design of a bolted joint is possible. Present results show, however, that the bolts in such a joint will work together, and that the ultimate strength of the bolted joint will be much higher than that of the riveted joint.[33b]

In summary, the authors have so far seen no evidence which is inconsistent with the explanation they have suggested. They agree with Mr. De Vries that many riveted joints are better than Study 1 of Fig. 7(a) would indicate. The specimens tested for this curve had a tension, shear, bearing ratio of about 1 - 0.84 - 1.74, and no clamping. This should be about the lower limit of the S-N curve for single lap joints where no clamping is involved and where the holes have not been damaged, and where the bearing-tension ratio is not greatly in excess of 1.50 or 1.75. Many riveted joints of these proportions will show greater strength than this; those with high clamping. However, since it is impossible to predict what joints will have high clamping in the rivets, particularly in the rivets at the ends of the connections, the authors feel that greater strength than Study 1 Curve is not to be counted on. For double lap joints, where the bearing is not too high, a fatigue strength considerably greater than the above may be expected. In any case, some safety factor should be provided. The authors feel that final specifications should be based upon critical examination of experimentally determined S-N curve, where each controlling variable is investigated separately.

33b. p. 223.

34. Comparative Behavior of Bolted and Riveted Joints, Frank Baron and Edward W. Larson.

35. Slip under Static Loads of Joints with High-Tensile Bolts, R. A. Hechtman, D. R. Young, A. G. Chin, and E. R. Savikko.

Discussion of
"FREQUENCY OF MAXIMUM WIND SPEEDS"

by H. C. S. Thom
(Proc. Sep. 539)

E. J. GUMBEL,[1]—Mr. Thom should be congratulated for applying a new tool, the theory of extreme values, to engineering problems where no other statistical method can be used. He is certainly right that the largest wind speeds should be analyzed with the help of this theory, a procedure also advocated recently by the National Bureau of Standards. His argument that the wind speeds are by necessity non-negative and that therefore the extremes too are limited to the left is irrefutable. He concludes that the second asymptotic distribution of extreme values should be used, because it is limited to the left, and not the first one, which is unlimited in both directions.

From a practical standpoint this argument is not as strong as it looks offhand. Many observations of a positive variate are usually and successfully analyzed by the normal distribution, which is unlimited in both directions. This procedure is legitimate provided that the probabilities for negative values (which do not exist) are so small, say of the order 10^{-7}, that their occurrence within the possible number of observations is not to be expected.

The same argument holds for extreme values. Here experience has shown that the first asymptotic distribution can successfully be used for the analysis of floods, although floods are positive variates. This may also hold for extreme windspeeds.

A logical difficulty in the use of the second asymptote is due to the fact that it possesses no moments. One cannot ask for the expected largest wind speed, i. e., for the mean which would be reached by increasing the number of observations. It is necessary to be satisfied with the knowledge of the most probable value. In a similar way, no standard deviation exists, and it must be replaced by a probable error. This leads to a practical difficulty. Since the theoretical moments do not exist, sample moments cannot be used for the estimation of the parameters. As was pointed out by Mr. Thom, the estimation of the parameters by the maximum-likelihood method is very difficult without the use of IBM equipment, which is not always available.

These considerations have led to the use of the first asymptote.(1) The probability F(x) of the largest value x is then

$$F(x) = \exp\left[-e^{-\alpha(x-u)}\right]$$

The parameters α and u can be very easily estimated from the sample mean $\overline{x}$ and sample standard deviation s with the help of

$$u = \overline{x} - \overline{y}_n/\alpha; \;\; 1/\alpha = s/\sigma_n$$

where $\overline{y}_n$ and σ_n are given as functions of the sample size n in a table published by the Bureau of Standards.(2) The calculation on the basis of Thom's

1. Columbia Univ., New York, N. Y.

data leads to $\bar{x} = 40.3333$; $s = 7.5996$. Since $\bar{y}_{39} = 0.5430$; $\sigma_{39} = 1.1388$ the parameters are $1/\alpha = 6.6734$; $u = 36.7097$. The theoretical values obtained from

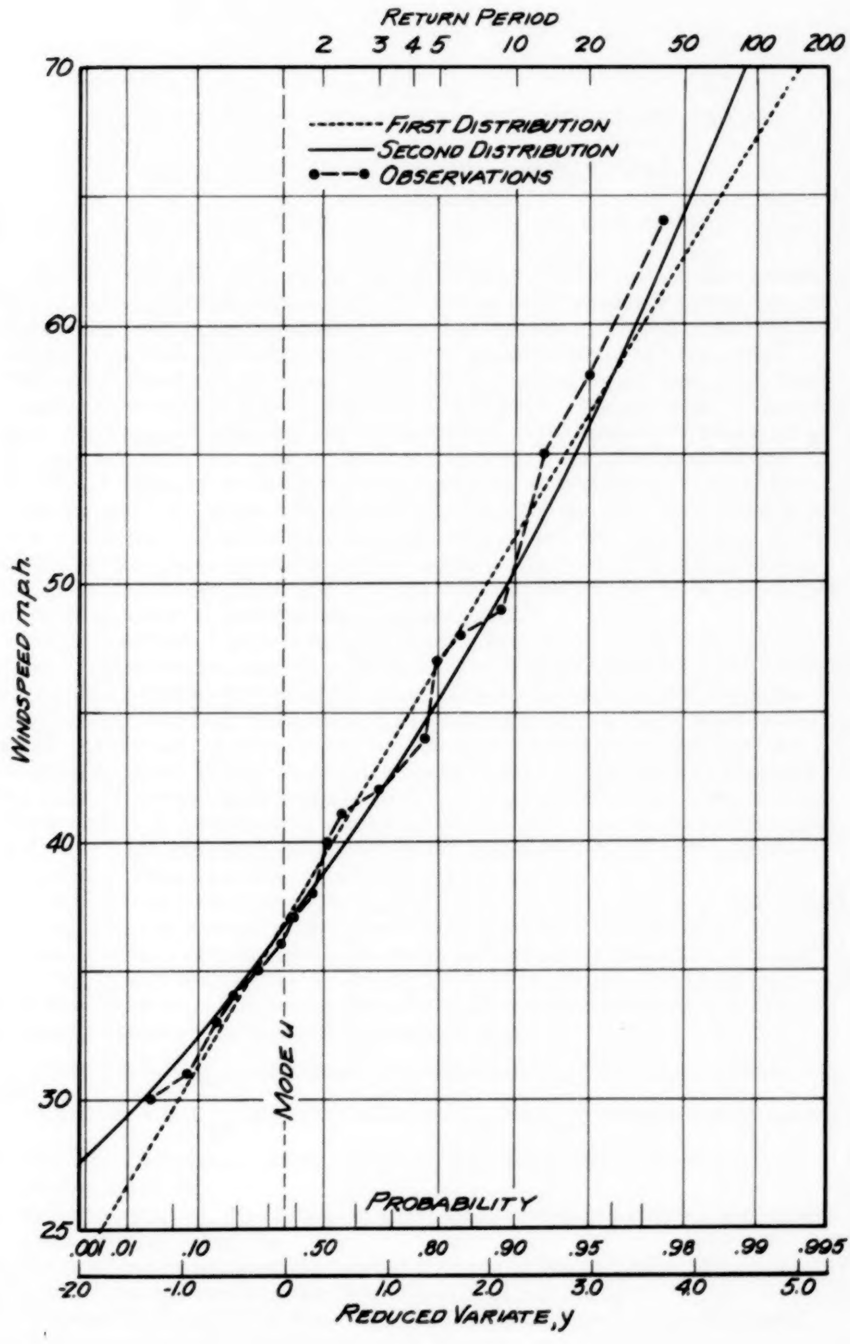
$$x = 36.71 + 6.67y$$

are traced together with the observations and values obtained by Thom in the graph. Extrapolation for the return periods 100 and 200 lead to wind speeds of 67.4 and 71.6 miles per hour from the first asymptote, while the second asymptote leads to slightly larger values 71.0 and 79.5. The differences between the two methods are not very large, and no graphical decision as to which of the two methods gives a better fit is possible.

A similar indecision will hold generally if the quotient of the observed maximum by the observed minimum of the largest value is only of the order 2, because then the plotting of the observed largest values will not be too different from the plotting of their logarithms. Only experience can show which of the two methods leads to a better fit. The logical superiority of the second asymptotic distribution is compensated by the lack of moments and the difficulties of calculating the parameters in all those cases where IBM equipment is not available.

REFERENCES

1. A. Court. Wind extremes as design factors. Franklin Inst., vol. 256, p. 39-56, Philadelphia, 1953.
2. E. J. Gumbel. Statistical theory of extreme values and some practical applications. Applied Mathematics Series No. 33, p. 31, Washington, 1954.


RETURN PERIOD
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FIRST DISTRIBUTION
SECOND DISTRIBUTION
OBSERVATIONS
WINDSPEED m.p.h.
MODE u
PROBABILITY
.001 .01
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.99
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Discussion of
"THE OCTAGONAL GIRDER FOUR COLUMN SPACE FRAME"

by P. C. Disario, J. S. Podolan and N. A. Weil
(Proc. Sep. 542)

SIDNEY SHORE,[1] J.M. ASCE, and HUMBERTO RINCON[2]—In the introduction to this paper it is stated that: "To the authors' knowledge, however, no study exists analyzing the stresses in space frames with horizontally curved members, subject to loading in, or normal to, their plane of curvature." There does appear in the literature methods that can be adapted, quite easily, to analyze the type of space frame being discussed. For example, if the members of the space frame are either segmental[3] in the horizontal plane, or of circular arc plan,[4] the moment distribution techniques offer avenues for solutions. Although the moment distribution solution usually is not of the "closed form", its versatility many times justifies its use. This appears to be particularly true in the case of supporting structures for chemical equipment and bins, since each type requires its own geometrical configuration. The so-called closed form is most advantageous when a series of structures of the same general configuration are designed.

To demonstrate a typical moment distribution solution, the same octagonal frame, solved numerically on page 542-23, will be analyzed for the distributed loading and columns hinged at the base. It is not feasible to use the same nomenclature and sign convention regarding moments as the authors used. In this discussion the sign convention for moments acting on the ends of a member is shown in Fig. 1b, and the nomenclature is that used in Separate No. 261.[3] The right hand screw rule will determine the sense of the moment vector, which is denoted by a double headed arrow. Fig. 1b, gives the values of the stiffness and carry-over factors for member 1-2, the only member skewed with respect to the reference axes, x, y, z.

Due to the symmetry of the octagonal girder and the uniformly distributed loading, only one quarter of the structure (A12B) has to be considered; further, the distribution about only one of the two coordinate axes, x and z (see Fig. 1a), is necessary. In Tables I and II the distribution about the x-axis only is shown since the carry-over moments from the z-axis to the x-axis are obtained by the following relationships,

$$\text{COM (Z)} = (+C_{xz})\,(\text{Distributed Moment})_z = (-C'_{zx})\,(\text{Distributed Moment})_x$$

and

$$\text{COM (Z)} = (+C'_{xz})\,(\text{Distributed Moment})_z = (-C_{zx})\,(\text{Distributed Moment})_x$$

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2. Graduate Student, The School of Civil Engineering, University of Pennsylvania, Philadelphia, Pa.
3. "Numerical Analysis of Continuous Frames in Space", by J. Michalos, ASCE Proceedings Separate No. 261, September 1953.
4. "Numerical Analysis of Frames with Curved Girders", by J. Michalos, ASCE Proceedings Separate No. 250, August 1953.

Essentially, the number of degrees of freedom that the segment A12B has is two. However, due to symmetry of loading and structure, the translation of both joints 1 and 2, perpendicular to plane x-z, will be identical. Therefore, Table I gives the end moments due to the uniformly distributed loading of 6.196 k/ft., with joints 1 and 2 restrained against translation. Table II presents the distribution of moments produced by giving joints 1 and 2 any arbitrary, but equal, displacements perpendicular to the x-z plane. In fact, these arbitrary displacements are of such a magnitude to produce fixed end moments of 100 ft. kips at the ends of members A-1 and 2-B. From the results of Table I and II, the final end moments were calculated from the conditions that in the actual structure there were no supports at joints 1 and 2.

The procedure for calculating the final end moments are shown in Fig. 2, and it can be seen that the moment distribution results agree excellently with the authors' solution (Table 7, page 542-25) at the locations of maximum moments. At those locations where the moments are rather small in magnitude, the discrepancies, percentage-wise, appear large, but this is of little consequence since the design of the member will not be based on these small values. Of course, closer correlation could have been obtained, if several more cycles of distribution were performed. However, it is felt that the technique has been adequately demonstrated by the 9 distribution cycles shown in Tables I and II.

The solution of the other two conditions of loading, i. e., horizontal shear loading and moment loading will not present any difficulties, the only variation being that the two degrees of freedom of the segment A12B will manifest itself under these loadings.

TABLE I. X-AXIS DISTRIBUTION

LOADING W = 6.196 K/FT. (NO JOINT TRANSLATION PERMITTED) FT-KIPS

JOINT	A			1		2		
MEMBER	AB	AA'	A1	1A	12	21	2B	B2
K/ΣK	0.361	0.278	0.361	0.387	0.613	0.23	0.77	—
C	$C'_{xx}=-1.0$	0	$C'_{xx}=-1.0$	$C'_{xx}=-1$	$C'_{zx}=.263$ $C'_{zx}=.579$ $C_{zx}=.686$	$C'_{zx}=.263$ $C'_{zx}=.579$ $C_{zx}=.686$	$C'_{xx}=.50$	—
F.E.M.					-27.85	+27.85	-9.90	+9.90
DIST. C.O.M_x C.O.M_z	-10.80		-10.80	+10.80	+17.05 -1.09 -9.85 +2.84	-4.15 +4.50 -11.70 +2.40	-13.80	-6.90
Σ[a]	-10.80		-10.80	+10.80	-18.90	+18.90	-23.70	+3.00
DIST. C.O.M_x C.O.M_z	+7.80 -3.14	+6.00	+7.80 -3.14	+3.14 -7.80	+4.96 +0.29 -2.87 -0.75	+1.10 +1.30 -3.40 -0.64	+3.70	+1.85
Σ	-6.14	+6.00	-6.14	+6.14	-17.27	+17.26	-20.00	+4.85
DIST. C.O.M_x C.O.M_z	+2.26 -4.30	+1.76	+2.26 -4.30	+4.30 -2.26	+6.83 +0.17 -3.95 -0.43	+0.63 +1.80 -4.70 -0.36	+2.11	+1.05
Σ	-8.18	+7.76	-8.18	+8.18	-14.65	+14.63	-17.89	+5.90
DIST. C.O.M_x C.O.M_z	+3.10 -2.50	+2.40	+3.10 -2.50	+2.50 -3.10	+3.97 +0.20 -2.30 -0.51	+0.75 +1.05 -2.72 -0.43	+2.51	+1.25
Σ	-7.58	+10.16	-7.58	+7.58	-13.29	+13.28	-15.38	+7.15
DIST. C.O.M_x C.O.M_z	+1.80 -2.21	+1.40	+1.80 -2.21	+2.21 -1.80	+3.50 +0.13 -2.02 -0.33	+0.43 +0.92 -2.40 -0.28	+1.62	+0.81
Σ	-7.99	+11.56	-7.99	+7.99	-12.01	+12.00	-13.76	+7.96
DIST. C.O.M_x C.O.M_z	+1.59 -1.55	+1.24	+1.59 -1.55	+1.55 -1.59	+2.47 +0.10 -1.43 -0.27	+0.40 +0.65 -1.70 -0.23	+1.36	+0.68
Σ	-7.95	+12.80	-7.95	+7.95	-11.14	+11.12	-12.40	+8.64
DIST. C.O.M_x C.O.M_z	+1.12 -1.23	+0.86	+1.12 -1.23	+1.23 -1.12	+1.96 +0.07 -1.13 -0.20	+0.29 +0.52 -1.34 -0.17	+0.99	+0.48
Σ	-8.06	+13.66	-8.06	+8.06	-10.44	+10.42	-11.41	+9.12
DIST. C.O.M_x C.O.M_z	+0.96 -0.92	+0.74	+0.96 -0.92	+0.92 -0.96	+1.46 +0.06 -0.85 -0.15	+0.22 +0.38 -1.00 -0.13	+0.77	+0.38
Σ	-8.02	+14.40	-8.02	+8.02	-9.92	+9.92	-10.64	+9.50
DIST.	+0.59	+0.46	+0.59	+0.74	+1.16	+0.17	+0.55	
F.M.[b]	-7.43	+14.86	-7.43	+8.76	-8.76	+10.09	-10.09	+9.50

a. Sub-Total of Moments
b. Final Moments

TABLE II. X-AXIS DISTRIBUTION

JOINT TRANSLATION ONLY ALL MOMENTS IN FT-KIPS

JOINT.	A			1		2		
MEMBER	AB	AA'	A1	1A	12	21	2B	B2
K/ΣK	0.361	0.278	0.361	0.387	0.613	0.23	0.77	—
C	$C'_{1x}=-1.0$	0	$C'_{1x}=-1.0$	$C'_{1x}=-1.0$	$C_{1x}=.263$ $C_{2x}=.579$ $C_{3x}=.686$	$C_{1x}=.263$ $C_{2x}=.579$ $C_{3x}=.686$	$C'_{1x}=0.50$	—
F.E.M.							+100	+100
DIST. C.O.M. C.O.M.					 -6.05 +15.80	-23.00 +13.30	-77.00	 -38.50
Σ					+9.75	-9.70	+23.00	+61.50
DIST. C.O.M. C.O.M.	 +3.77		 +3.77	-3.77	-5.98 -0.82 +3.45 +2.12	-3.18 -1.57 +4.10 +1.79	-10.20	 -5.10
Σ	+3.77		+3.77	-3.77	+8.52	-8.48	+12.80	+56.40
DIST. C.O.M. C.O.M.	-2.71 +1.84	-2.12	-2.71 +1.84	-1.84 +2.71	-2.91 -0.26 +1.68 +0.69	-1.00 -0.77 +2.00 +0.58	-3.32	 -1.66
Σ	+2.90	-2.12	+2.90	-2.90	+7.72	-7.67	+9.48	+54.74
DIST. C.O.M. C.O.M.	-1.33 +1.86	-1.02	-1.33 +1.86	-1.86 +1.33	-2.96 -0.11 +1.71 +0.29	-0.42 -0.78 +2.03 +0.24	-1.39	 -0.69
Σ	+3.43	-3.14	+3.43	-3.43	+6.65	-6.60	+8.09	+54.05
DIST. C.O.M. C.O.M.	-1.34 +1.24	-1.04	-1.34 +1.24	-1.24 +1.34	-1.98 -0.09 +1.14 +0.23	-0.34 -0.52 +1.36 +0.20	-1.15	 -0.57
Σ	+3.33	-4.18	+3.33	-3.33	+5.90	-5.90	+6.94	+53.48
DIST. C.O.M. C.O.M.	-0.90 +1.00	-0.68	-0.90 +1.00	-1.00 -0.06 +0.91 +0.16	-1.57 -0.41 +1.08 +0.14	-0.24	-0.80	 -0.40
Σ	+3.43	-4.86	+3.43	-3.43	+5.34	-5.33	+6.14	+53.08
DIST. C.O.M. C.O.M.	-0.72 +0.74	-0.56	-0.72 +0.74	-0.74 -0.05 +0.68 +0.13	-1.17 -0.31 +0.80 +0.11	-0.19	-0.62	 -0.31
Σ	+3.45	-5.42	+3.45	-3.45	+4.93	-4.92	+5.52	+52.77
DIST. C.O.M. C.O.M.	-0.53 +0.57	-0.42	-0.53 +0.57	-0.57 +0.53	-0.91 -0.04 +0.53 +0.10	-0.14 -0.24 +0.62 +0.08	-0.46	 -0.23
Σ	+3.49	-5.84	+3.49	-3.49	+4.61	-4.60	+5.06	+52.54
DIST	-0.42	-0.30	-0.42	-0.43	-0.69	-0.11	-0.35	
F.M.	+3.07	-6.14	+3.07	-3.92	+3.92	-4.71	+4.71	+52.54
p_x(F.M.)	+9.58	-19.17	+9.58	-12.22	+12.22	-14.70	+14.70	+169.10

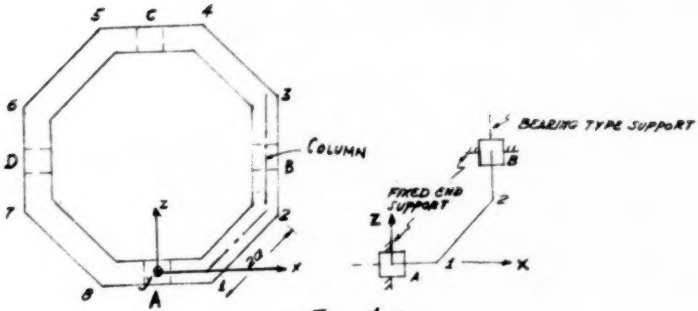
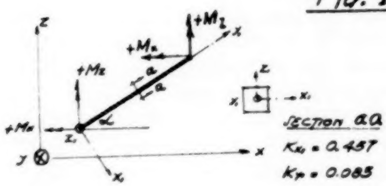


FIG. 1a



MEMBER 1-2

$\alpha = 45°$

$K_z = K_{y_1} \sin^2\alpha + K_{x_1} \cos^2\alpha = 0.271$

$K_x = K_{y_1} \cos^2\alpha + K_{x_1} \sin^2\alpha = 0.271$

$K_y = K_{z_1}$

$C_{xz} = \dfrac{(K_{x_1} - K_{y_1}) \sin\alpha \cos\alpha}{K_z} = 0.686$

$C_{zx} = \dfrac{(K_{x_1} - K_{y_1}) \sin\alpha \cos\alpha}{K_x} = 0.686$

$C_{yz} = C_{yx} = C_{zy} = C_{xy} = 0$

$C'_{xz} = (C_{12}^{x_1} + 1) \sin\alpha \cos\alpha + C_{xz} (C_{12}^{x_1} \sin^2\alpha - \cos^2\alpha) = 0.579$

$C'_{zz} = C_{12}^{x_1} \cos^2\alpha - \sin^2\alpha + C_{xz} \sin\alpha \cos\alpha (C_{12}^{x_1} + 1) = 0.263$

$C'_{xx} = C_{12}^{x_1} \sin^2\alpha - \cos^2\alpha + C_{zx} \sin\alpha \cos\alpha (C_{12}^{x_1} + 1) = 0.263$

$C'_{zx} = (C_{12}^{x_1} + 1) \sin\alpha \cos\alpha + C_{zx} (C_{12}^{x_1} \cos^2\alpha - \sin^2\alpha) = 0.579$

$C'_{yz} = 0 \qquad C'_{yx} = 0$

$C'_{zy} = 0 \qquad C'_{xy} = 0$

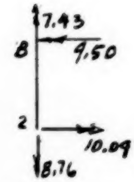
$C'_{yy} = C_{12}^{z_1} = 0.5$

NOTE: $C_{12}^{x_1} = C_{12}^{z_1} = 0.5$ SINCE MEMBER 1-2 IS PRISMATIC.

FIG. 1b

$$Y_1 = \left[\frac{(6.196)(4.367)}{2} + \frac{10.09 - 9.50}{4.367} \right] + \left[\frac{(6.196)(8.739)}{2} \right]$$

$$Y_1 = 40.85^K \uparrow$$



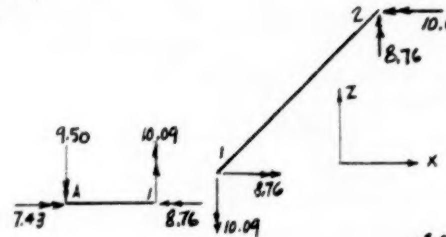
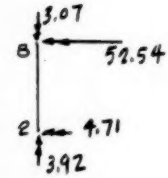


FIG 2a
W-LOADING ONLY
I



$$Y_1' = \frac{52.54 + 4.71}{4.367} = 13.10^K \downarrow$$

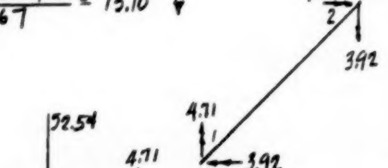
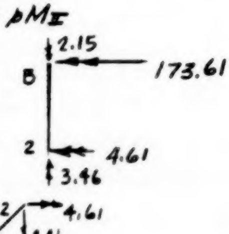


FIG 2b
JOINT TRANSLATION
ONLY
II

$$p = \frac{Y}{Y_1'} = \frac{40.85}{13.10} = 3.12$$

$$M_{FINAL} = M_I + pM_{II}$$



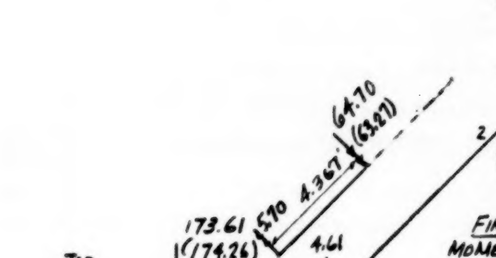


FIG 2c
FINAL MOMENTS IN FT KIPS
MOMENTS IN ( ) ARE THOSE LISTED
IN TABLE 7, p. 542-25.

Discussion of
"ANALYSIS OF THE VIERENDEEL GIRDER BY BALANCING THE PANEL MOMENTS"

by A. F. Diwan

W. D. BIGLER,[1] A.M. ASCE.—This solution of the Vierendeel girder deserves a place among the preferred mathematical procedures of structural engineering. No longer will it be necessary to treat the Vierendeel truss with sloping chords as a lengthy problem in simultaneous equations.

The author's method for computing the fixed-end moments is easily stated in a few words and the idea is easily applied: the points of inflection lie on a vertical line through the center of gravity of the area of the panel, and the vertical shear that is held by bending moment is distributed between the upper and lower chords in proportion to their K values. A trial problem solved by this method demonstrates that it works.

There is, however, a disadvantage that shows up in the odd-numbered lines of Table I, and that is the slowness of the correction for unbalanced vertical shears in the interior panels as the moment distribution progresses. This difficulty could easily be remedied by a slight change in procedure.

In panel DEMN, for example, the first correction for shear deficiency (line 3) totals 7360 ft. lbs., and the summation of all corrections in eight steps for this panel adds to 22,262 ft. lbs., or three times the original correction. Furthermore, it is apparent at a glance that after eight steps final convergence has not been reached, and it appears that the true values of the moments in the chords may be asymptotic.

For the seven panels, from left to right, the ratios of the total correction in the odd-numbered lines to the initial correction are 1.25, .55, 3.64, 3.02, 2.75, 2.19, and 1.62. The deviation from the general pattern of these values in the second and third panels is due to the sharp reversal of shear at C-L, caused by the concentration of the entire load at that point. This deviation would not exist under usual truss loading.

It is apparent from these seven ratios that slope in the chords tends to speed convergence. The reason for this is that the first trial moments in the chords, shown in line 1, are computed from the entire vertical shear, although, as shown by the introduction of the expression — H.b, each sloping chord actually holds a portion of the shear in direct stress. Thus there is an initial overestimate of the shear in those panels with sloping chords. If a similar aid of initial extra shear allotment was added in the interior bays, convergence would likewise be speeded up there.

In providing such initial over-correction a logical pattern of multipliers should be selected. For a Vierendeel truss of the proportions used in this example, probably a suitable pattern would be a multiplier of 2.0 for each computed correction in each step in the center panel, 1.8 in the third and fifth panels, 1.5 in the second and sixth, and 1.2 in the end panels. As the work progresses the designer can adjust these multipliers if he finds that a different value will lead more directly to convergence.

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W. A. KRAJEWSKI,[2]—In his very interesting paper concerning the analysis of a Vierendeel girder, Mr. A. T. Diwan states that there is "the difficulty in using the moment distribution method originally developed by Professor H. Cross". The moment distribution method is at present internationally accepted and it has been modified many times since it was originally presented by Professor H. Cross. Professor Dr. S. Blaszkowiak, of the Technical University of Gdansk, Poland, in his book "Metoda Cross'a" (Method of Cross)[3] applies moment distribution to 82 types of indeterminate structural shapes, some of them most complicated including the Vierendeel girder.

As a discussion, and to compare results of the two ways of solution for this same girder, the writer presents here the analysis of the truss shown in Fig. 1, which was previously analysed twice by Mr. Diwan. Fig. 1 shows dimensions, location of concentrated force and relative stiffnesses - same as Mr. Diwan's Fig. 4-a. For the distribution of moments the modified Hardy Cross method called by Professor Blaszkowiak "method of symmetry and antimetry" of sections was used. As the given Vierendeel girder has top and bottom chord sections with different moments of inertia and relative stiffness factors, we have to find an equivalent girder with the same sections for both chords. This is done simply by averaging the moments of inertia. Therefore, I average for both chords equals $\frac{I_{top} + I_{bott}}{2}$. As the rigidity of the chords in such a girder is the same and the shearing forces and bending moments on both chords act in the same direction we divide the shearing forces equally between the top and bottom chords and analyse one chord only. This is the case of the so called antimetry. However, the averaging of the moments of inertia produces an excess moment of inertia in one chord and lack of it in another chord. This discrepancy in per cent is equal to:

$$\epsilon = \frac{I_{top} - I_{bott}}{I_{top} + I_{bott}} \times 100.$$

Therefore, we must add in this same proportion moments on the side where we reduced the moment of inertia and deduct where we increased the moment of inertia. This produces the condition of loading symmetry (moments are of the same magnitude but act on each chord in an opposite direction) and for such condition the moments have to be distributed again. Then by using an equivalent girder with equal sections for top and bottom chords we created two cases: one of loading antimetry and one of loading symmetry. For both cases we find separately the fixed end moments, carry over factors and stiffness factors. For both cases we distribute the moments separately and considering only one chord. For antimetry the distributed moments on both chords are of the same sign and the same value, for symmetry the distributed moments on both chords have the same value but opposite sign. By adding respectively we obtain the final moments at all joints. To allow for such a shortcut in our calculations we have to calculate for the antimetry case special stiffness factors in the chord members and use 1.5 of standard relative stiffnesses for vertical members. For the symmetry case the standard relative stiffness of the verticals must be reduced by half. Actually, in the case of antimetry the most difficult part of the whole problem is to ascertain the stiffness factors and the fixed end moments. This is presented in a visual

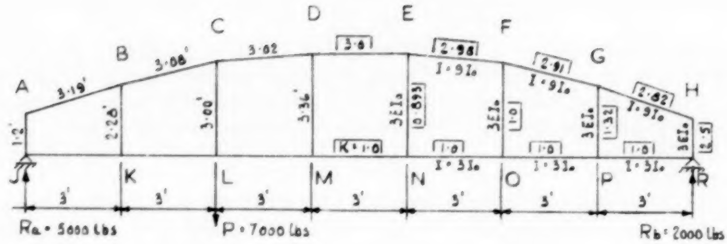
2. Structural Design Engineer, The Hydro Electric Power Commission of Ontario.
3. Metoda Cross'a, Professor Dr. S. Blaszkowiak, Gdansk, Poland, 1948, (in Polish language).

and self-explanatory way for the extreme left panel on Fig. 2. However, this kind of procedure would be a very lengthy one and because of this it is carried out in tabular form in Table 1 and 2. In calculating the stiffness factor, the formula presented on Fig. 3 was used but instead of ratio $\frac{b}{a}$ the writer employed the identical ratio $\frac{h_r}{h_{r \pm 1}} = \gamma$. The calculations for the fixed end moments are shown in Table 2 where γ is the aforementioned ratio, n is the number of bays to the left or right of the particular joint with the proper sign. For the fixed end moments there is given specially derived formula, where all values are known. The results from this formula agree with the diagrammetrical calculations on Fig. 2. A summary of the fixed moments is given on Fig. 4.

After the preparation of this data we distribute the moments on Table 3. This table is self-explanatory. Line 1 shows the location of girder members in relation to the joints, line 2 - upper chord relative stiffnesses, line 3 - bottom chord relative stiffnesses, line 4 - average stiffnesses, line 5 - results using the formula written at the top of the table showing in per cent the excess or lack of stiffness by making chords of average stiffness. Line 6 represents the value of stiffness factors for chords as calculated in Table 1 or stiffness factors of verticals from line 4 multiplied by 1.5. Line 7 gives the distribution factors in per cent. Line 8 - carry over factors obtained by multiplying the stiffness factors by their respective ratios γ from Table 1. Line 9 - these carry over factors in per cent of the sum of the distributing factors and line 10 - the fixed moments to be distributed as calculated in Table 2.

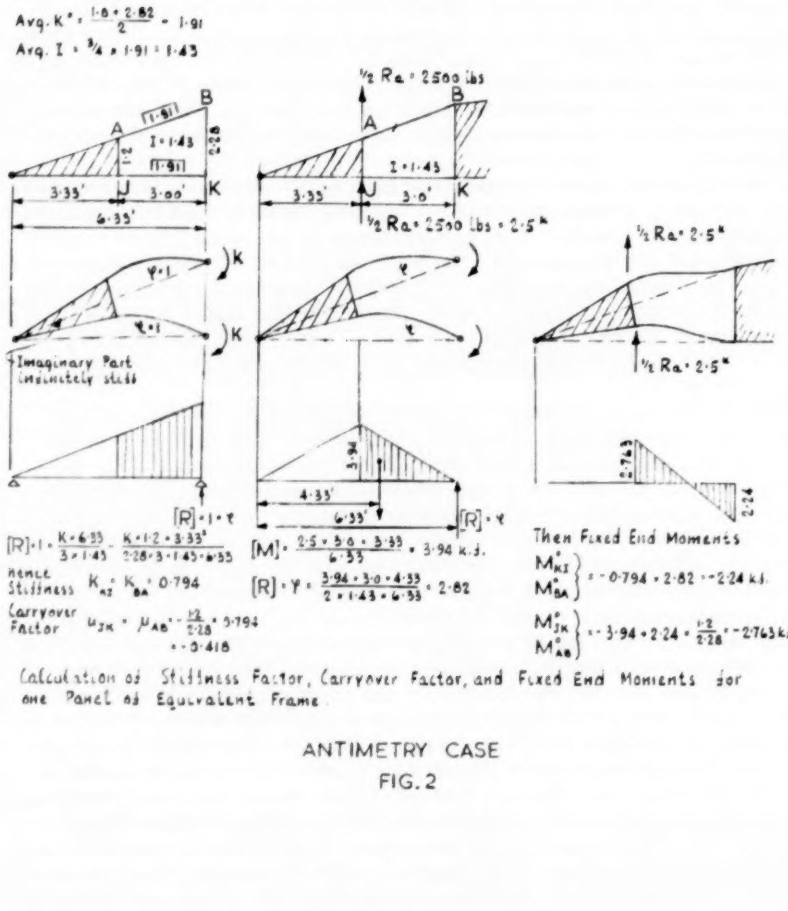
In the symmetry part of the table we have again: line 11 - stiffness factors as for symmetry system, that is line 4 but with the values for the verticals multiplied by 0.5, line 12 - distribution factors in per cent, line 13 - carry over factors, which are standard "halves" and line 14 - fixed end moments for the symmetry case (see also Fig. 5), which are obtained by multiplying the moments resulting from antimetry distribution by the discrepancy factor ϵ given in line 5 (for example -3221 x 0.47643 = -1535). At the bottom of the table is the summarization of moments from antimetry and symmetry cases (see also Fig. 6).

Table 4 shows a favourable comparison of results using the method described above with those obtained by Mr. Diwan. The difference in the moments between the distribution method and the exact method is on the average about 3 per cent which for practical purposes is sufficiently accurate. The writer does not think that more time is required for this method than that of Mr. Diwan. One thing is certain, moment distribution is more appealing to many engineers because of its clarity. In this case moment distribution was applied to a more unusual problem which had to be simplified before the standard Hardy Cross distribution was used. However, the difficulty of sloping chord could not be avoided and the stiffness factors and fixed end moments required a special method of calculation. The error would be smaller, if the discrepancy co-efficient was smaller. For $\epsilon \leq 1/3$, error is about 1 per cent according to Professor Dr. S. Blaszkowiak. In our case ϵ is nearly 0.5 and that is why the error is greater. Obviously, the girder with parallel chords would not take half as much time for the obtaining of the final moments. Mr. Diwan's method, shows clearly that such complicated structures as a Vierendeel girder may be solved comparatively easily. However, the writer believes that the moment distribution is somewhat clearer and it can be applied to almost any difficult problem like Vierendeel trusses, irregular frames, multiple arches or for the purpose of finding influence lines and secondary stresses.



DIMENSIONS & RELATIVE STIFFNESSES

FIG. 1



Calculation of Stiffness Factor, Carryover Factor, and Fixed End Moments for one Panel of Equivalent Frame

ANTIMETRY CASE

FIG. 2

Calculation of Stiffness Factor K for Top & Bottom Chords					
Chord Members		$\frac{h_r}{h_{r+1}} = \gamma$		$\frac{3/4\ K^\circ}{1+\gamma+\gamma^2}$	$= K$
A-B J-K	H-G R-P	$\frac{2 \cdot 28}{1 \cdot 2}$	1·90	$\frac{0 \cdot 75 \times 1 \cdot 91}{1 + 1 \cdot 90 + 1 \cdot 90^2}$	0·2200
B-C K-L	G-F P-O	$\frac{3 \cdot 0}{2 \cdot 28}$	1·315	$\frac{0 \cdot 75 \times 1 \cdot 955}{1 + 1 \cdot 315 + 1 \cdot 315^2}$	0·36256
C-D L-M	F-E O-N	$\frac{3 \cdot 36}{3 \cdot 0}$	1·120	$\frac{0 \cdot 75 \times 1 \cdot 99}{1 + 1 \cdot 12 + 1 \cdot 12^2}$	0·4423
D-E M-N	E-D N-M	$\frac{3 \cdot 36}{3 \cdot 36}$	1·00	$\frac{0 \cdot 75 \times 2 \cdot 0}{1 + 1 + 1^2}$	0·500
B-A K-J	G-H P-R	$\frac{1 \cdot 2}{2 \cdot 28}$	0·5265	$\frac{0 \cdot 75 \times 1 \cdot 91}{1 + 0 \cdot 5265 + 0 \cdot 5265^2}$	0·7942
C-B L-K	F-G O-P	$\frac{2 \cdot 28}{3 \cdot 00}$	0·760	$\frac{0 \cdot 75 \times 1 \cdot 955}{1 + 0 \cdot 76 + 0 \cdot 76^2}$	0·6272
D-C M-L	E-F N-O	$\frac{3 \cdot 0}{3 \cdot 36}$	0·89285	$\frac{0 \cdot 75 \times 1 \cdot 99}{1 + 0 \cdot 8929 + 0 \cdot 8929^2}$	0·5548
K° is average stiffness $\frac{I}{s}$ for Top Chord & Bottom Chord in the particular bay where s is the length of bay.					

TABLE I

$$M_A = \frac{3}{1 + \left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2} \times \frac{EI}{l}$$

$$M_B = -M_A \left(\frac{b}{a}\right)$$

FIG.3 Formula for Stiffness Factor and Carryover Factor; Beam with one end fixed and with hinge and infinitely stiff part at other end

Calculation of Fixed End Moments M° for P = 7000 #, Ra = 5000 #, Rb = 2000 #				
Chord Member	γ	n	$\frac{R \cdot s}{4} \times \frac{2n\gamma^2 - (n-2)\gamma - (n-1)}{1+\gamma+\gamma^2}$	$= M°$
A - B J - K	1·9000	0	$\frac{5000 \cdot 3}{4} \times \frac{0 + 2 \cdot 1{\cdot}90 + 1}{1 + 1{\cdot}90 + 1{\cdot}90^2}$	2765
B - C K - L	1·315	-1	$\frac{5000 \cdot 3}{4} \times \frac{-2 \cdot 1{\cdot}315^2 + 3 \times 1{\cdot}315 + 2}{1 + 1{\cdot}315 + 1{\cdot}315^2}$	2306
C - D L - M	1·120	+5	$\frac{2000 \times 3}{4} \cdot \frac{10 \times 1{\cdot}120^2 - 3 \times 1{\cdot}120 - 4}{1 + 1{\cdot}120 + 1{\cdot}120^2}$	2304
D - E M - N	1·000	+4	$\frac{2000 \times 3}{4} \times \frac{8 \times 1{\cdot}0^2 - 2 \times 1{\cdot}0 - 3}{1 + 1{\cdot}0 + 1{\cdot}0^2}$	1500
E - F N - O	0·893	+3	$\frac{2000 \cdot 3}{4} \times \frac{6 \times 0{\cdot}893^2 - 1 \times 0{\cdot}893 - 2}{1 + 0{\cdot}893 + 0{\cdot}893^2}$	1055
F - G O - P	0·760	+2	$\frac{2000 \times 3}{4} \times \frac{4 \times 0{\cdot}760^2 - 0 \times 0{\cdot}760 - 1}{1 + 0{\cdot}760 + 0{\cdot}760^2}$	841
G - H P - R	0·5265	+1	$\frac{2000 \cdot 3}{4} \times \frac{2 \times 0{\cdot}5265^2 + 1 \times 0{\cdot}5265 - 0}{1 + 0{\cdot}5265 + 0{\cdot}5265^2}$	899
B - A K - J	0·5265	+1	$\frac{5000 \times 3}{4} \cdot \frac{2 \times 0{\cdot}5265^2 + 1 \times 0{\cdot}5265 - 0}{1 + 0{\cdot}5265 + 0{\cdot}5265^2}$	2247
C - B L - K	0·760	+2	$\frac{5000 \cdot 3}{4} \times \frac{4 \times 0{\cdot}760^2 - 0 \times 0{\cdot}760 - 1}{1 + 0{\cdot}760 + 0{\cdot}760^2}$	2102
D - C M - L	0·893	-4	$\frac{2000 \cdot 3}{4} \times \frac{-8 \times 0{\cdot}893^2 + 6 \times 0{\cdot}893 + 5}{1 + 0{\cdot}893 + 0{\cdot}893^2}$	2218
E - D N - M	1·000	-3	$\frac{2000 \cdot 3}{4} \times \frac{-6 \times 1{\cdot}0^2 + 5 \times 1{\cdot}0 + 4}{1 + 1{\cdot}0 + 1{\cdot}0^2}$	1500
F - E O - N	1·120	-2	$\frac{2000 \cdot 3}{4} \times \frac{-4 \times 1{\cdot}12^2 + 4 \times 1{\cdot}12 + 3}{1 + 1{\cdot}12 + 1{\cdot}12^2}$	1095
G - F P - O	1·315	-1	$\frac{2000 \cdot 3}{4} \times \frac{-2 \times 1{\cdot}315^2 + 3 \times 1{\cdot}315 + 2}{1 + 1{\cdot}315 + 1{\cdot}315^2}$	922
H - G R - P	1·900	0	$\frac{2000 \cdot 3}{4} \times \frac{0 + 2 \times 1{\cdot}90 + 1}{1 + 1{\cdot}90 + 1{\cdot}90^2}$	1106

TABLE 2

$\varepsilon = \frac{\text{Upp.} - \text{Bott.}}{\text{Upp.} + \text{Bott.}} \cdot 100$

	JOINT J			JOINT K				JOINT L				JOINT M				JOINT N				JOINT O				JOINT P				JOINT R		
1		J-A	J-K		K-J	K-B	K-L		L-K	L-C	L-M		M-L	M-D	M-N		N-M	N-E	N-O		O-N	O-F	O-P		P-O	P-G	P-R		R-P	R-H
2	Up.	2.5	2.82		2.82	1.32	2.91		2.91	1.00	2.98		2.98	0.893	3.00		3.00	0.893	2.98		2.98	1.00	2.91		2.91	1.32	2.82		2.82	2.50
3	Bott.	2.5	1.00		1.00	1.32	1.00		1.00	1.00	1.00		1.00	0.893	1.00		1.00	0.893	1.00		1.00	1.00	1.00		1.00	1.32	1.00		1.00	2.50
4	Avg.	2.5	1.91		1.91	1.32	1.955		1.955	1.00	1.99		1.99	0.893	2.00		2.00	0.893	1.99		1.99	1.00	1.955		1.955	1.32	1.91		1.91	2.50
5	E%		47.643		47.643		48.849		48.849		49.749		49.749		50.000		50.000		49.749		49.749		48.849		48.849		47.643		47.643	
	ANTIMETRY																													
6	3.97	3.75	0.220	3.1368	0.7942	1.98	0.3626	2.5695	0.6272	1.50	0.4423	2.3943	0.8548	1.3395	0.500	2.3943	0.500	1.3395	0.5548	2.5695	0.4423	1.50	0.6272	3.1368	0.3626	1.98	0.7942	3.97	0.220	3.75
7	100%	94.46	5.54	100%	26.32	63.12	8.56	100%	24.41	58.38	17.21	100%	23.18	55.95	20.88	100%	20.88	55.95	23.18	100%	17.21	58.38	24.41	100%	8.56	63.12	26.32	100%	5.54	94.46
8	c.o. factor		0.4181		0.4181		0.4768		0.4768		0.4954		0.4954		0.500		0.500		0.4954		0.4954		0.4768		0.4768		0.4181		0.4181	
9	%		10.53		13.33		15.20		18.55		19.28		20.70		20.88		20.88		20.70		19.28		18.56		15.20		13.33		10.53	
10	-3372		+2765	-4553	-2247		-2306	+280	-2162		+2304	+3718	+2218		+1500	+3704	+1500		+1065	+1936	+1095		+841	+2296	+922		+899	+1106	+1106	
			-607						-692		+770						+776		+373						+359		+116			
				-312	-355		+43					+827	+54		+773					+1116	+767		+349					+306	+306	
	-42		-42					+124	-47		+171					+388	+173		+215					+239	+207		+32			
				+19	-4		+23					+105	+24		+81					+116	+80		+36					+32	+32	
	+3		+3					+26	+3		+22					+44	+22		+22					+26	+22		+3			
				+5	0		+5					+14	+5		+9					+15	+9		+4					+3	+3	
	+1		+1					+5	+2		+3					+6	+3		+3					+2	+2		0			
				+1	0		+1					+2	+1		+1					+1	+1		0					0	0	
	+3410	+3221	+189	+4840	+1226	+3055	+560	-434	-106	-255	-75	-4666	-1082	-2810	-974	-4142	-365	-2317	-960	-5182	-548	-1858	-777	-2562	-296	-1617	-649	-1466	-80	-1366
	0.0	+3221	-3221	0.0	-1381	+3055	-1674	0.0	-2942	-253	+3195	0.0	+1220	-2610	+1390	0.0	+1609	-2317	+708	0.0	+1404	-1858	+453	0.0	+1216	-1617	+401	0.0	+1367	-1366
	SYMMETRY																													
11	3.16	1.25	1.91	4.525	1.91	0.66	1.955	4.445	1.955	0.50	1.99	4.487	1.99	0.497	2.00	4.487	2.00	0.497	1.99	4.445	1.99	0.50	1.955	4.525	1.955	0.61	1.91	3.16	1.91	1.25
12	100%	39.55	60.45	100%	42.21	14.59	43.20	100%	43.98	11.25	44.77	100%	44.35	11.08	44.57	100%	44.57	11.08	44.35	100%	44.77	11.25	43.98	100%	43.20	14.59	42.21	100%	60.45	39.55
13	c.o. factor		30.225		21.115		21.60		21.49		22.385		22.175		22.285		22.285		22.175		22.385		21.49		21.60		21.115		30.225	
14	-1223		-1535	-1476	-658		-818	+182	-1457		+1589	+1302	+607		+695	+661	+805		+352	+919	+698		+221	+391	+894		+191	+651	+651	
			+312						+319		-289						-290		-206						-197		-197			
				+331	+370		-39					-188	-41		-147					-231	-147		-84					-83	-83	
	-70		-70					-29	-71		+42					+94	+42		+52					+75	+50		+25			
				+27	+21		+6					-15	+6		-21					-37	-21		-16					-16	-16	
	-6		-6					-3	-6		+3					+11	+3		+8					+13	+8		+5			
				+3	+2		+1					-1	+1		-2					-5	-2		-3					-3	-3	
	-1		-1					-1	-1		0					+1	0		+1					+2	+1		+1			
	+1300	+514	-786	+815	+471	+163	+482	-149	-66	-17	-67	-1098	-487	-122	-489	-767	-342	-85	-340	-646	-289	-73	-284	-481	-268	-78	-203	-549	-332	-217
	0.0	±514	±514	0.0	±207	±163	±370	0.0	±1262	±17	±1279	0.0	±86	±122	±36	0.0	±218	±85	±133	0.0	±239	±73	±166	0.0	±248	±78	±178	0.0	±217	±217
	SUM																													
	Upper Chord	+3735	-3735		-1174	+3218	-2044		-4204	-270	+4474		+1306	-2732	+1426		+1827	-2402	+575		+1643	-1931	+287		+1464	-1687	+223		+1584	-1584
	Bottom Chord	+2707	-2707		-1588	+2892	-1304		-1680	-236	+1916		+1134	-2488	+1354		+1391	-2252	+841		+1165	-1785	+619		+968	-1547	+579		+1150	-1150

TABLE 3
MOMENT DISTRIBUTION

Member	Relaxation Method	Exact Method	Hardy Cross as used by Blaszkowiak	Percent Error to Exact Method
A - B	-3835	-3832	-3735	-2.5%
B - A	-1240	-1208	-1174	-2.8%
B - C	-1950	-1980	-2044	+3.1%
C - B	-4210	-4340	-4204	-3.1%
C - D	+4470	+4580	+4474	-2.3%
D - C	+1222	+1266	+1306	+3.0%
D - E	+1400	+1446	+1426	-1.3%
E - D	+1792	+1846	+1827	-1.0%
E - F	+531	+572	+575	+0.5%
F - E	+1635	+1676	+1643	-1.9%
F - G	+275	+266	+287	+7.1%
G - F	+1515	+1500	+1464	-2.3%
G - H	+200	+218	+223	+2.2%
H - G	+1635	+1633	+1584	-3.0%
J - K	-2646	-2633	-2707	+3.0%
K - J	-1545	-1532	-1588	+3.4%
K - L	-1245	-1231	-1304	+5.5%
L - K	-1674	-1695	-1680	-1.0%
L - M	+1868	+1911	+1916	+0.2%
M - L	+1070	+1077	+1134	+5.5%
M - N	+1294	+1329	+1354	+2.0%
N - M	+1347	+1377	+1391	+1.2%
N - O	+790	+822	+841	+2.1%
O - N	+1120	+1154	+1165	+0.9%
O - P	+595	+592	+619	+4.3%
P - O	+965	+961	+968	+0.5%
P - R	+555	+537	+579	+7.7%
R - P	+1122	+1125	+1150	+2.1%

COMPARISON OF RESULTS

TABLE 4

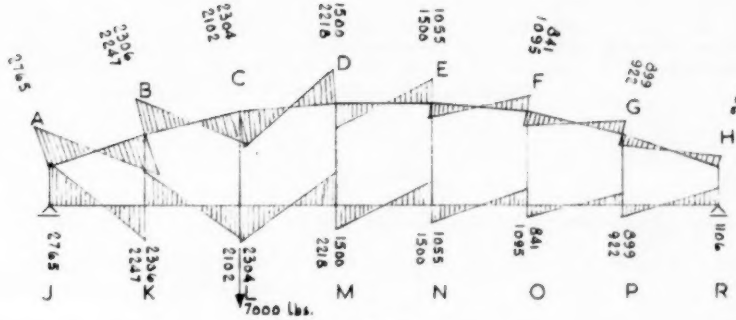


FIG.4

FIXED END MOMENT DIAGRAM-ANTIMETRY CASE

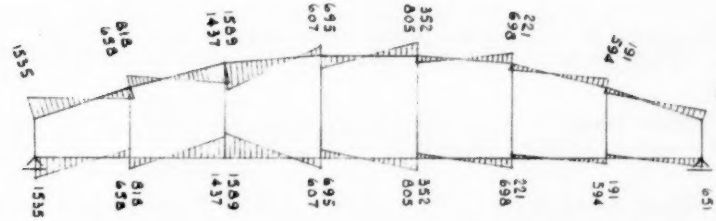


FIG. 5

FIXED END MOMENT DIAGRAM - SYMMETRY CASE

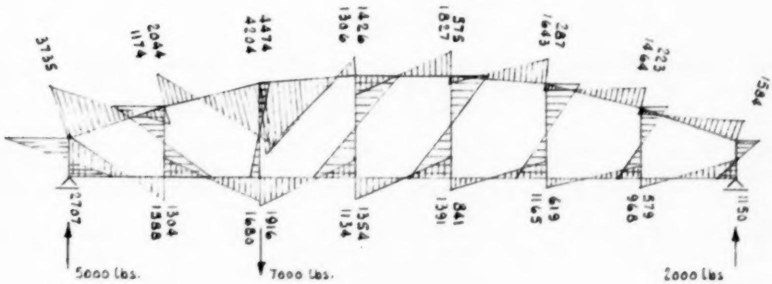


FIG.6

FINAL MOMENT DIAGRAM

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

MAY: 679(ST), 680(ST), 681(ST), 682(ST)[c], 683(ST), 684(ST), 685(SA), 686(SA), 687(SA), 688(SA), 689(SA)[c], 690(EM), 691(EM), 692(EM), 693(EM), 694(EM), 695(EM), 696(PO), 697(PO), 698(SA), 699(PO)[c], 700(PO), 701(ST)[c].

c. Discussion of several papers, grouped by Divisions.

e. Presented at the Atlantic City (N.J.) Convention in June, 1954.